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NEW METHODS OF LOAD-CARRYING CAPACITY PREDICTION FOR THE ULTIMATELY COMPRESSED FRAME STRUCTURES

Amid acute problems that arise in the field of rocket and space technology, mechanical engineering, and other fields and require a workable engineering solution, the problem of prediction and prevention of the unpredicted collapse of the structural members of the structures subjected to loading is considered. Prediction of the load-carrying capacity and residual life of the space frames during the long-term operation is based on the analysis of the stress and strain state, using readings from the strain and displacement pickups installed in the most loaded zones. In this case the yield strength of the structural material or the fatigue strength of the material may be considered as the criterion of the maximum load. At the same time the loss of stability of the compressed structural members used in the load-carrying thin-walled structures are among the potentially dangerous failure modes. In these cases such failure occurs unexpectedly without any visible signs of change in the initial geometry. Application of the adequate diagnostic techniques and methods of prediction of the maximum loads under compression conditions will make it possible to avoid the structural failures. In this case an assembly under test may be used for other purposes. To perform static strength testing, the rocket and space companies use costly compartments of as-built dimension. Therefore, keeping compartments safe solves an important problem of saving financial costs for hardware production. Nowadays this problem is particularly acute when ground testing the new technology prototypes.

Key words: space frames, load-carrying members, stress and strain state, loss of stability, prediction of the structural failure.

Серед актуальних проблем у ракетно-космічній техніці, а також у сучасному машинобудуванні та в інших галузях, що потребують практичного інженерного вирішення, розглядають прогнозування та запобігання незапланованому руйнуванню силових елементів навантажених конструкцій і споруд. Прогнозування несучої здатності й остаточного ресурсу просторових конструкцій під час тривалої експлуатації в цей час ґрунтується на аналізі напружено-деформованого стану з використанням показань датчиків деформації та датчиків переміщень у найбільш навантажених зонах. У такому разі як критерій гранично допустимого навантаження можна розглядати границю плинності конструкційного матеріалу або границю утоми матеріалу. Разом з тим до характерних видів потенційно небезпечного руйнування належить втрата стійкості стиснених силових елементів, використовуваних у несучих тонкостінних конструкціях. Руйнування в таких випадках відбувається раптово, з відсутністю видимих ознак зміни вихідної геометричної форми. Застосування достовірних методів діагностики та способів прогнозування гранично допустимих навантажень в умовах стиснення дозволить під час міцнісних випробувань не призводити конструкцію до руйнування. У такому разі видається можливим використовувати випробуване складання для інших цілей. У ракетно-космічній техніці для статичних випробувань на міцність використовують дорогі відсіки натурних розмірів. Тому збереження відсіків цілими вирішує важливе завдання економії фінансових витрат на виготовлення матеріальної частини. У цей час ця проблема особливо актуальна під час наземного відпрацювання зразків нової техніки.

Ключові слова: просторові конструкції, силові елементи, напружено-деформований стан, втрата стійкості, прогнозування руйнування конструкції.

Introduction

The authors of the work [1] emphasize the extraordinary complexity of loss of stability of shell structures. On this matter the expression by D. Bushnell, a famous American scientist in the field of deformable systems mechanics, is cited, who called loss of stability the «pitfall for designers». C. Truesdell, another American scientist who is known for his works in the field of the continuum mechanics, also noted on this matter «When we try, however, to investigate stability in general, it turns out that not only is it hard to investigate, but primarily it is hard to define it accurately»[1] ([2], p. 350). On the merits of the problem, the viewpoints of American and National scientists are in line. Yu. Rabotnov pointed out that the known

[1] Translation from Russian.

classical Euler-Lagrange approach «…settles a problem not of stability in the literal sense of word but of a possibility of existence of the two different forms of balance at the same value of the force» ([3], p. 120). V. Bolotin had the same opinion [4]: «…Euler's method in Theory of Elastic Stability does not contain, as a matter of fact, stability… existence of a branch point is neither necessary nor sufficient condition of change of stability». An appropriate physical and mathematical model has not been developed so far, because the mechanism of loss of stability has not been adequately defined yet. The trusted proactive prediction of the thin-walled structure failure requires a stability analysis to be theoretically well-founded with no empirical corrections. *Longstanding practice of using the empirical coefficients in the engineering stability analysis holds back the development of ideas on the physical nature of stability loss.*

From the conceptual standpoint, another argument in favor of performing a complete analysis of the stability problem is the fact that it has been more than 270 years now since Euler, in 1744, developed for the first time the classical formula $T^E = \frac{\pi^2 EI}{L^2}$ to define the buckling compressive force of a straight bar of L length, while bar ends are pivot joined (where E – modulus of elasticity of the structural material, I – moment of inertia of the cross section of the bar in the plane of the lowest bending stiffness). However, a theoretical solution has not been obtained neither for an elongated straight column with an end support (Fig. 1), nor for other essentially important engineering problems arising due to loss of stability *in the large*. A distinctive feature of loss of stability in the large is a spontaneous occurrence of the transverse mechanical impulse that *in the absence of imposed transverse force provides the sudden crippling of the extremely compressed elastic structural members that is followed by final deflection and dynamic effect.* The elongated straight metal column with flat ends without connections given in Fig. 1 can serve as a clear and powerful example of dynamic effect with final deflection at the time of sudden crippling. Such columns under compression conditions find practical application for erection and construction operations.

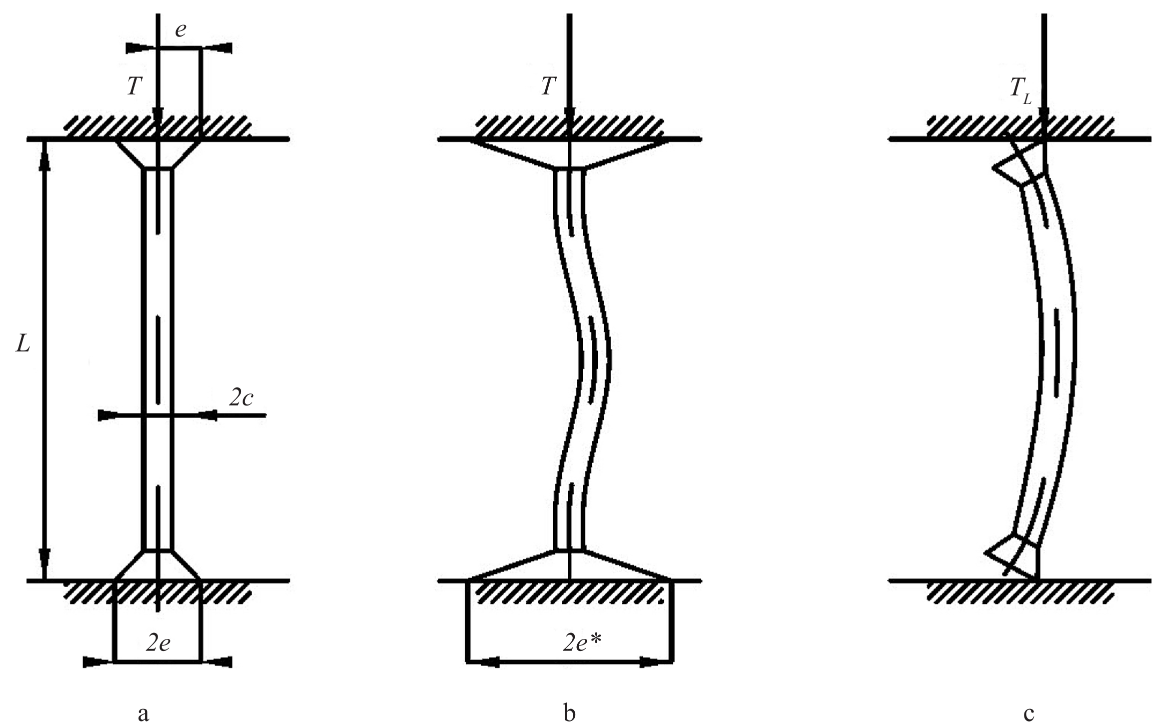

Fig. 1. The diagram of the support column under load: a – an initial position of the column in the state of subcritical compression; b – a supposed bending form after the loss of stability in the small (with no dynamic effect); c – loss of stability in the large (with dynamic effect)

The column with elastic properties set between two stiffened plates and loaded with axial compressive force T by means of a movable upper or bottom plate is under consideration. The column may be subjected to axial compression between two fixed plates, providing uniform heating. In this case it necessitates solving the stability problem of the column taking into account the thermal deformation. The examples illustrate the columns ends, which have an extended support area with dimension of $2e$ in the plane of the Figure. The thickness of the normal profile of the column in the same plane of the Figure is characterized by the parameter $h = 2c$. In the plane, perpendicular to the Figure, the adequate dimensions of the column profile and the support area are characterized by the much greater value of b which provides crippling of the elastic column in the plane of the Figure. Thus the cross section of the column under consideration is square and characterized by the thickness of $h = 2c$ and width of b, ($b >> 2c$).

Problem statement

The technical and scientific books on mechanics of deformable systems do not present a theoretical method for determining the critical load for the considered case of axial loading applied to the elastic support column of regular geometry. Due to lack of the theoretical justification for loss of stability *in the large* in the ultimately compressed straight bar with flat ends, there is no an appropriate design formula to determine the critical value of the axial force to be used in the engineering practice.

To solve the problem of stability *in the small*, the Euler formulation allows the occurrence for one reason or another of the small deflections $w(x)$, which provides the use of a homogeneous differential equation of the column bending in the plane of the smallest stiffness (in the plane of Fig. 1).

$$EI\frac{d^4w}{dx^4} + T\frac{d^2w}{dx^2} = 0 . \qquad (1)$$

Arbitrarily small deflections can presumably be considered at a loss of stability of the column according to the diagram given in Fig. 1, b, which would correspond to the stiff restraint of the ends. With the suitable boundary condition, the solution of the quoted differential equation determines the value of the critical force $T^*_{cr} = \frac{4\pi^2 EI}{L^2}$, that is four times the value of the Euler force T^E with pivoted connection of the ends. If stability is lost according to the diagram in Fig. 1, c, than differential equation (1) to determine the moment of the column sudden crippling is *not applicable*, since the restoring moment of the finite quantity T^E holds back the small deflections in the bending form. However, from the standpoint of classical mechanics [5], the column with an end support has to cripple according to the diagram in Fig. 1, b, and hence cannot lose its stability at the axial force value less than $T^*_{cr} = \frac{4\pi^2 EI}{L^2}$, which would contradict the experimental data. Under actual conditions, the sudden crippling of the considered metal column with the end support that have limited linear dimensions occurs according to the diagram in Fig. 1, c at the axial force value $T^E < T_{cr} < T^*_{cr}$. The results of the static tests of axial compression of the elongated frame samples with flat ends are shown below.

To obtain the experimental data for the loading diagram under consideration, using the sheet of rolled steel, material – Steel 20, the static test laboratory produced the flat frame samples of the regular geometry. The samples had square cross section with width of $b = 50$ mm. The level of actual irregularities was defined by the technological procedure during the manufacture of the steel sheet. The visual signs of the samples deviation from the original straight form were not found. The elongated samples of the first type had the *pivoted support ends* under the axial compression. For that reason the ends of the samples were beveled that is they had sharp edges ($e = 0$, Fig. 1, a). The first type samples had the following parameters: $L = 450$ mm, $b = 50$ mm; $2c = h = 8.25$ mm. The material elasticity modulus was $E = 2.1 \cdot 10^6$ kg/cm^2. In this case the design value of Euler's force was $T^E = 2390$ kgf. The results obtained under the static tests of the two samples showed the following values of critical force: 2537 kgf; 2574 kgf. The axial force was measured by the strain gauge force sensor TB-25. The axial displacements of the loaded bar ends were measured by the strain gauge displacement sensor ДП-10.

Slight differences between the experimental values of critical force and the design value of Euler's force ~10 % (6 % and 8 %) prove the adequacy of the stability calculation method in the Euler formulation (loss of stability in the small) with the pivoted support of the straight bar ends, and in the case of the pivoted support of the elongated metal column as well. The calculation method with the loss of stability in the small in the Euler-Lagrange formulation is widely used to calculate the critical load for thin-walled members, including shell structures. A relevant value of the critical load for the shell is referred to as «upper» critical load. Under actual conditions when the static load is applied to the structures of as-built dimensions with cylindrical and spherical shells that have actual irregularities, loss of stability in the large occurs at much less (three–four times) axial compressive loads in comparison with the upper critical load. Thus, as noted above, the empiric coefficients are used in the engineering practice of the stability analysis. To implement loss of stability in the large with the dynamic effect of buckling in terms of the one-dimensional problem, the two samples of the second type with the same geometrical dimensions as the first type were additionally manufactured. Yet the ends of the straight samples were flat with the dimensions of $2e = h$, ($b = 50$ mm). Therefore the opportunity offered the axial compression by the stiffened plates according to the diagram shown in Fig. 1, a. When the samples with the flat ends lost their stability, the deflections were formed according to the diagram in Fig. 1, c. According to the test data, the following values of the critical axial force were obtained for the two samples: 5447 kgf and 6478 kgf. The excess ratio of the critical axial force value in comparison with Euler force $T^E = 2390$ kgf at the pivoted support in this case is 2.28 and 2.7, respectively. The third type of the sample was manufactured of greater length ($L = 540$ mm) and extended support area at the end with the parameter $2e = 12.3$ mm, ($2e = 1.5h$, $b = 50$ mm). The thickness of the cross section: $h = 8.3$ mm. The axial force was loaded continuously by the movable stiffened plate up to the sudden buckling at the crippling of the bar.

As a result the sample took the curved shape according to Fig. 1, c. Loss of stability resulted at the axial force $T_{cr} = 4400$ kgf that was 2.59 times greater than the design value of Euler force ($T^E = 1700$ kgf). Such an essential difference of the critical force under the end support condition from the Euler force value at the pivoted support requires theoretical substantiation. At the same time, the axial force value $T_{cr} = 4400$ kgf is significantly lower than the design value of critical force $T^*_{cr} = 6800$ kgf at the rigidly restrained ends of the sample bar under consideration.

The technical books that introduce some researches in the field of strength and stability do not substantiate the mechanism of crippling at loss of stability in the large. Analyzing the compressive force loaded to the straight bar with the flat end, V. Feodosyev, well-known specialist in the field of stability of elastic system states: «The proposed problem touches upon fundamentally new problems on stability and cannot be solved by standard methods» ([5], p. 260). To make sure of it, the author studies the design diagram of transverse-longitudinal bending, where from the very beginning of loading the moment T^E, where $e = c$, was applied to the bent bar. However, the problem with such formulation does not obtain the required result, because «it is impossible to catch a critical transition from the rectilinear form of equilibrium to the curvilinear form of equilibrium». In practice of the theoretical calculation of shell structures the «method of non-ideality» is used, where moment subcritical state in the nonlinear formulation is taken into account. This method also does not help solving the problems of stability in the large, among which there is a problem of loss of stability of the support column according to the diagram in Fig. 1, c. Since in this case the considered column has no initial deflection, the sudden crippling occurs in the absence of the moment subcritical state.

The analysis in terms of the one-dimensional problem with allowance for the experimental data proves that the stability problem can be solved by finding out *the physical nature of the sudden crippling with transverse dynamic impulse in the absence of the external transverse force*. The article [6] contains the investigations on how the initial irregularities level affects the intensity of the sudden crippling of the ultimately compressed bar at the given boundary conditions according to the classical diagram: the pivoted support, rigid restraint of the ends. It was theoretically established that in this case *the intensity of the transverse* *mechanical impulse* and the appropriate value of the pitch of the macro deflection with loss of stability of the compressed elastic bar are defined by *the elastic deformation energy of the bending* accumulated within the subcritical phase of loading due to the irregularities.

1. Mechanism of the transverse mechanical impulse formation with loss of stability in the large

With the loss of stability of the elastic straight bar with the flat ends, loaded with the axial force by the stiffened plate according to the diagram in Fig. 1, the final deflections that occur at the sudden crippling of the bar in accordance with the diagram in Fig. 1, c are considered. This process of spontaneous formation of deflections is characterized by loss of stability in the large and is followed by the transverse dynamic impulse – «buckling».

It has been noted before that *the physical nature of the sudden crippling with the transverse dynamic impulse in the absence of the imposed transverse force is still unknown.*

With the Euler force $T^E = \dfrac{\pi^2 EI}{L^2}$, the considered column and the bar with the flat ends under the axial compression are still straight, which is proved by the experiments conducted on the prototypes. In this case the sufficiently small deflections $w(x)$ are prevented by the restoring moment of finite quantity T^E (Fig. 1, c). In the process of spontaneous crippling the considered bar shall go through the intermediate position, where there is no static equilibrium. Therefore, it is impossible to determine the coordinates of the space position of the bar particles in such conditions. Essentially, it means that the coordinate location of the particles at the sufficiently small deflections within the considered process of the spontaneous crippling of the ultimately compressed bar remains undefined. As a result the *Heisenberg uncertainty principle* is implemented at the macro level. The quantum mechanics deals with elementary particles at the microlevel in the state of uncertainty. According to the uncertainty principle *«the coordinates and momentum of particle cannot simultaneously take on accurate values*». Taking into account the analysis it can be concluded that the ultimately compressed bar with flat ends in the process of sudden crippling in the state of uncertainty assumes the wave properties at the macro level. In this case, according to the quantum mechanics, in order to provide the spontaneous crippling of the bar with the loss of stability in the large, the *energy – momentum* relation is required.

With certain value of the axial force $T_{cr} > T^E$, the ultimately compressed bar with flat ends loses stability and forms the finite deflections with amplitude of the order of the sample thickness *h*. From the energy standpoint, before crippling takes place, the bar assumes potential energy of the subcritical compression characterized by *threshold level* $Th^* = \dfrac{T_{cr}^2 L}{2EF}$. In the closing phase of the sudden crippling, the bar shifts from its uncertainty state to the new position of static equilibrium in the bent form (Fig. 1, c). In this case, the static equilibrium is provided in the condition of the pivoted support of the flat ends of the bent bar, as it is shown in Fig. 1, c. After crippling, the axial force action is determined by the lower value which equals to Euler force $T_l = \dfrac{\pi^2 EI}{L^2}$ that corresponds to the proper value of the axial force potential energy $Th_l = \dfrac{T_l^2 L}{2EF}$. According to the classical solution [7], the deflection under such conditions is characterized by a half wave sine curve $w^* = f^* \sin\dfrac{\pi x}{L}$. The design diagram of the static equilibrium at the pivoted connection is implemented with the value of deflection amplitude of $f^* \geq h = 2c$. In such conditions, the action of the restoring moment T^E is eliminated.

The position of the static equilibrium of the bar in the bent state is characterized by *the formative energy of the elastic deformation of the bend*. To determine the formative energy of the bend, the classical formula is used $E_f = \dfrac{1}{2} EI \int_0^L \left(\dfrac{d^2 w}{dx^2} \right)^2 dx$. As follows from the analysis the energy of bending deformation E_f must be created by the transverse mechanical impulse, required to overcome *the energy hump*, caused in this case by the restoring moment $(T_{cr})e$. It requires the relevant energy that in this process is determined by the relation $\Delta Th_0^* = \dfrac{L}{2EF}\left[\left(T_{cr}\right)^2 - \left(T_l\right)^2 \right]$.

It should be noted that the mechanism of transforming the potential energy ΔTh_0^* into the transverse mechanical impulse requires the physical substantiation. The articles [8], [9] contain the proper physical substantiation in terms of *the symmetry principle of physical phenomena* that deals with the idea of the *intrinsic space* of the deformable system. Taking elastic bar under the axial force compression as an example, the scientific and technical books for the first time theoretically established *the phenomenon of initiation of bending within the bar's eigenspace*. Eigenspace bending of the ultimately compressed elastic bar as *«a trigger»* is a crucial factor in the spontaneous occurrence of the transverse mechanical impulse when stability is lost in the large. The energy balance in this process corresponds to the classical energy conservation law according to the equation

$$\frac{L}{2EF}\left[\left(T_{cr}\right)^2-\left(\frac{\pi^2 EI}{L^2}\right)^2\right]=\frac{1}{2}EI\int_0^L\left(\frac{d^2w}{dx^2}\right)^2 dx. \quad (2)$$

The right part of the equation (2) indicates the formative energy of the bend E_f. In integrating the right part, the above expression for the bend $w^* = f^*\sin\frac{\pi x}{L}$ is used. In the static equilibrium of the bent bar with the flat end, the minimum value of the formative energy at which the amplitude of the specified bend is determined by the bar thickness $f^* = h$ is considered. Taking into account the assumed indication, the right part of the equation (2) in the result of the integration is expressed as $\frac{\pi^4 EIh^2}{4L^3}$. After the proper transformations the axial force T_{cr} in the equation (2) is considered as an unknown parameter. In this case the critical value of the axial force of the ultimately compressed bar with the flat end is determined by the formula

$$T_{cr}^* = \frac{\pi^2 EI}{L^2}\sqrt{\frac{h^2}{2i^2}+1}. \quad (3)$$

Here $i=\sqrt{I/F}$ – radius of inertia of the cross section of the bar with $F = bh$ area.

The formula (3) was used for the analysis of the results of the static stability tests of the bar prototypes under the axial compression. Since the cross section of the samples was square-formed with the radius of inertia of $i=\frac{h}{2\sqrt{3}}$, the formula (3) in this case transforms into $T_{cr}^* = 2.65T^E$.

Coefficient $k = 3$ is used in the specified expression to determine the critical value of the axial force for the bar, column with round cross-section.

The samples with square cross-section were manufactured in various forms of the end support:

1. To eliminate the influence of the end support and provide the pivoted support under the loss of stability of the prototype, the ends of the samples were beveled ($e = 0$). As follows from the conducted analysis, the critical value of the axial force in accordance with the experimental data in this case conforms to the design value that equals the value of Euler force $\frac{\pi^2 EI}{L^2}$ with pivoted joint of the ends of the centrally-compressed bar with length of $L_0 = 450$ mm, $L_0 = 540$ mm. The axial force was loaded in vertical and horizontal positions with no support of the prototypes on the horizontal base.

2. The ends of the prototypes are made flat ($2e = h$) with length of $L_0 = 450$ mm. The results of the static stability tests under the axial compression of prototypes with thickness of h in quantity of 9 pieces are presented in Table 1. The limiting value of axial force according to the experimental data is designated by the parameter T_{af}^{li}. The critical value of the axial force is designated by the parameter T_{cr}^* and calculated according to the formula (3). The degree of conformity of the experimental data with the theoretical value of critical force is characterized by the coefficient of stability $k=\frac{T_{af}^{li}}{T_{cr}^*}$ presented in Table 1.

The length change of the compressed prototype in accordance with test results is designated by the parameter ΔL. The theoretical value of the initial length change of the ultimately compressed prototype with the regular geometry and length of $L_0 = 45.0$ cm is calculated from the formula $\Delta L_0 = \frac{T_{af}^{li} L_0}{EF}$, where $E = 2.1 \cdot 10^6$ kg/cm^2 is the material modulus of elasticity, $F = bh$ is the area of the cross section of the prototype with the width of $b = 5.0$ cm.

3. The flat ends of the bars are made with the extended support pad (Fig. 1) in two options: $2e = 1.5h$, $2e = 2.7h$. The critical value of the axial force is designated by the parameter T^*_{cr} and calculated from the formula (3). The results of the static stability of the prototypes with the width h in quantity of 8 pieces are presented in Table 2. The Table uses designations assumed in Table 1. The prototypes in quantity of 6 pieces (items 1–6) with length of $L_0 = 450$ mm. The two prototypes (items 7–8, Table 2) with width $h = 8$ mm with the initial length of $L_0 = 540$ mm.

Table 1

Loading the prototypes with flat ends with compression axial force T_{af} ($2e = h$, Fig. 1)

No	h, mm	T^{li}_{af}, tonforce	ΔL, mm	ΔL_0, mm	$k = \dfrac{T^{li}_{af}}{T^*_{cr}}$	Note
1	7.7	5.1	0.42	0.28	0.99	Vertical loading $L_0 = 45$ cm
2	7.8	5.68	0.55	0.3	1.1	
3	7.8	5.19	0.50	0.3	1.0	
4	7.7	4.53	0.96	0.28	0.88	Horizontal loading with no support on the plane $L_0 = 45$ cm
5	7.8	4.94	0.97	0.27	0.95	
6	7.8	4.43	1.02	0.24	0.86	
7	8.25	6.48	1.4	0.33	0.98	
8	8.25	5.45	1.33	0.28	0.86	
9	8.25	6.03	1.4	0.31	0.95	

Table 2

Loading the prototypes with the extended support pads of $2e$ size with the compression axial force T_{af} (Fig. 1)

No	h, mm	T^{li}_{af}, tonforce	ΔL, mm	ΔL_0, mm	$k = \dfrac{T^{li}_{af}}{T^*_{cr}}$	Note
1	3.6	0.485	0.43	0.06	0.92	$2e = 10$ mm
2	3.6	0.487	0.7	0.06	0.92	$2e = 15$ mm
3	7.8	4.57	0.68	0.25	0.88	$2e = 12.3$ mm
4	7.8	4.9	0.52	0.27	0.95	$2e = 12.3$ mm
5	7.8	5.0	0.49	0.27	0.97	$2e =21.9$ mm
6	8.0	5.25	0.65	0.28	0.90	$2e = 21.9$ mm
7	8.3	4.4	0.9	0.27	0.98	$2e =12.3$ mm
8	8.3	5.3	0.7	0.33	1.18	$2e = 21.9$ mm

The analysis results, presented in Tables 1 and 2, indicate the conformity of the experimental values of the axial force at the sudden crippling of the ultimately compressed elastic prototypes with the theoretical value of the critical force, calculated by the formula (3).

The crucial factor at the crippling of the prototypes with extended support pad is the prototype thickness h used in the design formula (3). The upsizing of the support pad characterized by the parameter $e > h$ (Fig. 1) *hardly leads to the increase in the critical value of the axial force*, which conforms to Saint-Venant's principle in the edge effect analysis in the strength of material method. The maximum difference between the experimental data and the design values of the axial force under the loss of stability of the prototypes is 12–14 % downward axial force and 10–18 % upward axial force. Under the static loading of the full-scale compartment load-bearing members by the rocketry and other vehicles structures, the value of the stability coefficient k is within the scatter band ±15 %.

This section presents the solution of the loss of stability problem in the large in one-dimensional formulation based on the example of the elastic metal bar with flat ends. The energy criterion deals with the inner energy that is a certain part of the potential energy of the subcritical compression. The same approach should be used for analyzing loss of stability in the small of the centrally compressed straight bar, the pivoted ends of which are in fixed position during crippling. This case is considered in the engineering analysis practice under the thermal deformation of the heated metal bar.

2. Mechanism of deflections formation in the problem of stability of the straight bar subjected to heating

The below research deals with the following feature: under the loss of stability in the small without the dynamic effect, *the crippling with small macro deflection along the main bend shape can be provided not only by the continued loading of the external force T, but in some isolated cases due to the axial deformation caused by the thermal effect when heating the bar*. The crippling of the elastic bar with the length of L under the uniform heating up to the specific temperature can be considered as an example of «an isolated case». The pivoted ends connected to the completely stiffened and fixed supports [7] is under consideration.

The problem of stability of the heated bar is solved with the use of the homogenous differential equation (1) taking into account the internal force of compression $N(t) = T$ due to the thermal deformation in the absence of the axial displacement of the ends. According to the Euler–Lagrange concept, it is suggested that the centrally compressed straight bar made of homogeneous material with elastic properties obtains arbitrarily small deflection $w(x)$ unexpectedly occuring by any reason. The problem of stability in the small, using in this case the differential equation of static equilibrium (1) is solved with such assumption in the classical formulation. The classical approach also suggests that the crippling is accompanied with the continuous loading by the *external force T* that works along the axial displacement of the bar end. The work of force T along the axial displacement of the end provides the bend deformation of material fibers and the appropriate deflection of the bar. When the deflection of the bar $w(x)$ occurs, the work of the external force T transforms into the energy of the bend elastic deformation determined by the formula $U = \frac{1}{2} EI \int_0^L \left(\frac{d^2 w}{dx^2} \right)^2 dx$. *Under the loss of stability of the heated bar with the pivoted ends fixed, the fundamentally different formation mechanism for the energy of the bend elastic deformation is realized*.

For the first time the articles [6, 8, 10] established that the crucial factor of arbitrarily small deflections $w(x)$ occurring under the loss of stability of the thin-walled structures are weak disturbances at the level of the ambient noise (the vibrations of atmospheric air, acoustical pressure, emanating from weak sources of noise, weak vibrations of a loader). In conditions of small unforeseen environmental effects, the initiation of the arbitrarily macro deflections in the ultimately compressed deformable system becomes possible at zero value of the unit structural bending stiffness, that is to say under the «zero stiffness effect». While the pivoted support of the elastic bar the appropriate stiffness [6] with virtual deflections $\delta w_n(x) = \delta f_n \sin \frac{n\pi x}{L}$ with number of half-waves n in the subcritical loading phase is defined by the expression $C_n = \frac{n^2\pi^2}{L^2} \left(\frac{n^2\pi^2 EI}{L^2} - N \right)$. Parameter N as the axial force is determined by the mechanical stress of the axial compression σ_0 according to the relation $N = \sigma_0 F$. $At \cdot C_n = 0$ the actual deflection $w(x) = f \sin \frac{\pi x}{L}$, as the most probable one with the amplitude f of one half-wave of the sine curve ($n = 1$), corresponds to the minimum value of the parameter N.

To realize the possible, virtual deflection $\delta w(x)$ with the main bend shape into the actual arbitrarily small deflection $w(x)$, the potential energy of the axial deformation is used under the thermal effect during the heating of the elastic bar.

In the case under consideration the potential energy of the subcritical compression is determined by the internal force $N = E_t F \alpha \Delta t$ *with temperature increase by* Δt degrees with the linear expansion coefficient α. Value of the modulus of elasticity E_t is defined, taking into account the maximum attained temperature during the bar heating. The elongation of the heated bar in the condition of fixed supports is ruled out. However, with the temperature increase the potential energy of the subcritical compression is created in the elastic bar. For the straight metal bar with length of L and cross section area F, the value of the relevant energy P of the subcritical compression is determined by the expression $P = \frac{1}{2}\alpha^2 \Delta t^2 E_t FL.$

It was established above that the occurrence of the arbitrarily small deflection $w(x) = f \sin\frac{\pi x}{L}$ is possible in the «zero stiffness effect» of the ultimately compressed bar. In the process of crippling the ultimately compressed bar gets length increment, determined by the formula $\lambda = \frac{1}{2}\int_0^L \left(\frac{dw}{dx}\right)^2 dx$.

As a result of integration the increment λ is written down as $\lambda = \frac{\pi^2}{4L} f^2$ characterized by the value of the second order of vanishing. The length increment of the bar causes *the degradation of potential energy of compression*. In this case the corresponding energy degradation of compression is determined by the value of the second order of vanishing according to the expression $\delta P = P\lambda$. The considered axial force P (which equals to the internal force N) acts in the end sections of the heated bar fixed supports.

At the same time, it is necessary to take into account the energy of the elastic deformations of the bend $U = \frac{1}{2} E_t I \int_0^L \left(\frac{d^2 w}{dx^2}\right)^2 dx$ that is caused by the deflection $w(x) = f \sin\frac{\pi x}{L}$. As a result of integration the bending energy is written down as $U = \frac{\pi^4 E_t I}{4L^3} f^2$ that is also determined by the value of the second order of vanishing. According to the energy conservation law, the use of the equality $P\lambda = U$ is based on the energy principle. The energy balance in such process corresponds to the equation

$$P\frac{\pi^2}{4L} f^2 = \frac{\pi^4 E_t I}{4L^3} f^2. \qquad (4)$$

From the equation (4) follows the expression of the axial force P that determines the critical force of the axial compression that equals to the classical value of Euler force $T^E = \frac{\pi^2 E_t I}{L^2}$. Since the equality $P = N$ is true, where $N = E_t F \alpha \Delta t$, the maximum value of the temperature increment under the loss of stability of the heated bar with the pivoted support is determined from the assumption $E_t F \alpha \Delta t = \frac{\pi^2 E_t I}{L^2}$. As a result the relevant formula is written down as

$$\max \Delta t = \frac{\pi^2 I}{\alpha L^2 F}. \qquad (5)$$

The observation and experience indicate that the crippling of the uniformly heated bar occurs in the absence of the transverse dynamic impulse, which is typical for the loss of stability in the small.

As follows from the conducted studies on the stability of the column with flat ends, the formula to determine the limiting value of the temperature increment of the heated column can be expressed in the similar manner

$$\max \Delta t = \frac{\pi^2 I}{\alpha L^2 F}\sqrt{\frac{h^2}{2i^2} + 1}. \qquad (6)$$

Conclusions

The mathematic model approximation of the load-carrying capacity prediction of the ultimately compressed structural members in the frames is stated. For the first time in the field of mechanics of deformable systems the problem of stability in the large, characterized by the sudden formation of the deflections in finite quantities with mechanical impulse, has been solved on the basis of the nonstandard approach. The theoretically grounded design formulae have been obtained to determine the critical force under the axial compression of the column and bar with flat ends under the normal temperature and during heating up to the limiting temperature.

Using the prototypes under the normal temperature, the critical values of the axial force have been found for two options of support pads on the flat ends. The experimental data, shown in Tables, conform satisfactorily to the design values of critical force of the axial compression in the elastic region.

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