

УДК 629.78

G. A. Ivanova, Candidate of Science (physics and mathematics);
V. S. Khoroshilov, Doctor of Engineering Sciences

ON CONTROL OF SPACECRAFT ORIENTATION TO THE GROUND DATA ACQUISITION STATION

The article dwells on the spacecraft attitude control to point the onboard antenna to the ground data acquisition station during the communication session. Antenna is fixed relatively the spacecraft body. Purpose of the antenna is to receive the flight task aboard the spacecraft and to downlink the telemetry information. When orbiting, the spacecraft position relatively the ground data acquisition station changes continuously. It is due to the diurnal rotation of the Earth, spacecraft orbital motion and angular motion of the spacecraft relatively the center of mass under the impact of the disturbing and control moments. To tilt the spacecraft uses reaction wheels, installed in axes of coordinate system coupled with spacecraft center of mass. Electromagnets are used to unload the reaction wheels. The reaction wheels control law is suggested, which tilts the spacecraft to point the antenna to the ground data acquisition station. Mathematical model of the spacecraft dynamics relatively center of mass is given, using the suggested reaction wheels control law. The following external disturbing moments, acting on the spacecraft in flight, are taken into consideration: gravitational, magnetic, aerodynamic moments and solar radiation moment of forces. Dipole model of the magnetic field of the Earth is used to calculate the magnetic moments. Software was developed and spacecraft dynamics was simulated on the personal computer with the specified initial data. Simulation initial conditions correspond to the attitude control mode of the spacecraft relatively the orbital coordinate system with the specified accuracy. Simulation results verify the applicability of the suggested reaction wheel control law.

Key words: electrical axis of the antenna, mathematical model, coordinate system, transfer matrix, vector.

Розглянуто питання керування орієнтацією космічного апарата для наведення бортової антени на наземну станцію приймання даних протягом сеансу зв'язку. Антена нерухома відносно корпусу космічного апарата. Призначення антени – отримання на борт космічного апарата польотного завдання та скидання на Землю телеметричної інформації. Під час руху космічного апарата по орбіті його положення відносно наземної станції приймання даних безперервно змінюється. Це пов'язано з добовим обертанням Землі, рухом космічного апарата по орбіті та кутовим рухом космічного апарата відносно центру мас під дією збурних і керувальних моментів. Для повороту космічного апарата використовують двигуни-маховики, установлені по осях зв'язаної з центром мас космічного апарата системи координат. Для розвантаження двигунів-маховиків використовують електромагніти. Запропоновано закон керування двигунами-маховиками, що забезпечує поворот космічного апарата для наведення антени на наземну станцію приймання даних. Наведено математичну модель динаміки космічного апарата відносно центру мас з урахуванням запропонованого закону керування двигунами-маховиками. Із зовнішніх збурних моментів, що діють на космічний апарат у польоті, урахують гравітаційний, магнітний, аеродинамічний моменти та момент сил сонячної радіації. Під час обчислення магнітних моментів використовують дипольну модель магнітного поля Землі. Розроблено програму та проведено моделювання динаміки космічного апарата на персональному комп'ютері для заданих вихідних даних. Початкові умови під час моделювання відповідають режиму орієнтації космічного апарата відносно орбітальної системи координат із заданою точністю. Результати моделювання підтверджують застосовність запропонованого закону керування двигунами-маховиками.

Ключові слова: електрична вісь антени, математична модель, система координат, матриця переходу, вектор.

Problem formulation

Under consideration is the spacecraft (SC), which control system includes three reaction wheels (RW) and three electromagnets (EM), installed in the axes of the coordinate system coupled with the SC center of mass.

The problem is formulated as follows: spacecraft is turned to provide the pointing of the electric axis of the onboard transceiver antenna

to the ground data acquisition station (GDAS) in order to downlink the telemetry data.

While orbiting, spacecraft orbital position relatively the GDAS changes continuously. It is due to the diurnal rotation of the Earth, spacecraft orbital and angular motion relatively the center of mass under the impact of the disturbing and control moments.

Let's consider the problem of pointing the antenna's electric axis to the GDAS, turning the SC body. Fig. 1 shows the position of the

center of the Earth, O_z ; center of mass of the SC, O_c and location of GDAS, O_p

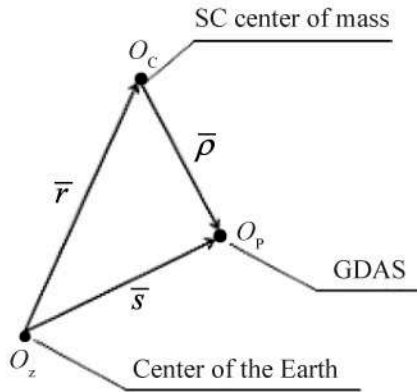


Fig. 1. Location of center of the Earth, SC center of mass and GDAS site

Vectors in the Fig. 1 signify the following:

\vec{r} – radius-vector, drawn from the center of the Earth to the SC center of mass and specifies the current position of the SC center of mass in orbit;

$\vec{\rho}$ – radius-vector, drawn from the SC center of mass to the point on the earth surface where GDAS is;

\vec{s} – radius-vector, drawn from the center of the Earth to the point where the GDAS is.

Vectors, shown in Fig. 1 are connected by relationship

$$\vec{r} + \vec{\rho} = \vec{s}. \quad (1)$$

The problem is reduced to obtaining the RW control law, providing the coincidence of the antenna's electric axis with the unit vector, directed along the radius-vector $\vec{\rho}$. Control law is checked by SC dynamics simulation relatively the center of mass.

Системы координат и матрицы перехода между ними

Developing the SC mathematical model, the following right-hand orthogonal coordinate

– matrix of transfer from ICS to OCS

$$T_{OI} = T_u T_i T_\Omega = \begin{bmatrix} \cos(u) & 0 & -\sin(u) \\ 0 & 1 & 0 \\ \sin(u) & 0 & \cos(u) \end{bmatrix} \cdot \begin{bmatrix} \cos(i) & \sin(i) & 0 \\ -\sin(i) & \cos(i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\Omega) & 0 & -\sin(\Omega) \\ 0 & 1 & 0 \\ \sin(\Omega) & 0 & \cos(\Omega) \end{bmatrix}, \quad (2)$$

where u – argument of a latitude; i – inclination of the orbital plane to the equatorial plane; Ω – longitude of the orbit's ascending node;

systems are used (position of the coordinate systems origin is shown in Fig. 1):

$O_z X_1 Y_1 Z_1$ – inertial coordinate system (ICS), axis $O_z Y_1$ is directed along the Earth's rotation axis towards the North Pole, axis $O_z Z_1$ – towards the vernal equinoctial point;

$O_z X_G Y_G Z_G$ – Greenwich coordinate system (GCS), axis $O_z Z_G$ is directed along the line of intersection of the Greenwich meridian and equatorial planes, axis $O_z Y_G$ is perpendicular to the equatorial plane and directed towards the North Pole;

$O_p X_p Y_p Z_p$ – coordinate system fixed with ground station (PCS), axis $O_p Z_p$ is directed along the radius-vector, connecting center of the Earth with GDAS, axis $O_p Y_p$ – tangentially towards the geographic meridian, passes through GDAS;

$O_c X_o Y_o Z_o$ – orbital coordinate system (OCS), axis $O_c Z_o$ is directed along the radius-vector, connecting the Earth's center with SC center of mass, axis $O_c X_o$ is in the plane of the SC orbit and is directed towards its orbital motion;

$O_c X_B Y_B Z_B$ – body-axis coordinate system (BCS), with ideal SC attitude BCS axes are parallel and codirectional with the respective OCS axes, when SC deviates from the ideal attitude position of BCS relatively the OCS is defined by the successive rotation by the pitch, roll and yaw angles;

$O_D X_D Y_D Z_D$ – design coordinate system (DCS), its origin O_D is in the SC/LV mating plane, DCS axes are parallel and codirectional with the respective BCS axes.

Above descriptions of the coordinate systems define the directions of two axes, the third axis in all cases completes the system to the right-handed one.

For the above coordinate systems, the following transfer matrices are introduced:

– matrix of transfer from OCS to BCS

$$T_{BO} = T_{\psi} T_{\varphi} T_{\theta} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\varphi) \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}, \quad (3)$$

where ψ, φ, θ – SC attitude angles in yaw, roll and pitch, respectively;

– matrix of transfer from ICS to GCS

$$T_{GI} = T_{\lambda} = \begin{bmatrix} \cos(\lambda) & 0 & -\sin(\lambda) \\ 0 & 1 & 0 \\ \sin(\lambda) & 0 & \cos(\lambda) \end{bmatrix}, \quad (4)$$

where $\lambda = \lambda_0 + \omega_z t$ – angular distance from vernal equinox direction to Greenwich meridian; λ_0 – initial value of the angle λ ; $\omega_z = 7,29211 \cdot 10^{-5} \text{ c}^{-1}$ – angular velocity of the diurnal rotation of the Earth;

– matrix of transfer from GCS to PCS

$$T_{PG} = T_{\varphi_{\text{geo}}} T_{\lambda_{\text{geo}}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi_{\text{geo}}) & -\sin(\varphi_{\text{geo}}) \\ 0 & \sin(\varphi_{\text{geo}}) & \cos(\varphi_{\text{geo}}) \end{bmatrix} \begin{bmatrix} \cos(\lambda_{\text{geo}}) & 0 & -\sin(\lambda_{\text{geo}}) \\ 0 & 1 & 0 \\ \sin(\lambda_{\text{geo}}) & 0 & \cos(\lambda_{\text{geo}}) \end{bmatrix}, \quad (5)$$

where $\lambda_{\text{geo}}, \varphi_{\text{geo}}$ – geographic longitude and latitude of the GDAS, respectively.

Inferior letter indices in the transfer matrices (2)–(5) correspond to the first letter in the acronym of the coordinate system, for example: I – ICS, O – OCS, B – BCK etc.

Formulas to calculate the current values of the orbit parameters u и Ω , included in (2), are given in [1].

SC antenna pointing to the GDAS

It follows from Fig. 1 that before starting the data downlink, SC has to orient the electric axis of its antenna along the vector $\bar{\rho}$, defined according to (1) as follows

$$\bar{\rho} = \bar{s} - \bar{r}. \quad (6)$$

Vector \bar{s} , represented in PCS, has the following form

$$\bar{s}^P = \begin{bmatrix} 0 & 0 & R_z \end{bmatrix}^T, \quad (7)$$

where $R_z = 6371 \text{ km}$ – mean radius of the Earth.

Vector \bar{r} , represented in OCS, has the following form

$$\bar{r}^O = \begin{bmatrix} 0 & 0 & r \end{bmatrix}^T, \quad (8)$$

where r – distance from the center of the Earth to the SC center of mass.

Equations (7), (8) and further as the text goes, the upper index at the vector corresponds to the first letter in the acronym of the coordinate system.

To determine the vector modulus $\bar{\rho}$ according to formula (6), it is required to transfer vector \bar{s}^P from PCS to OCS using the following expression

$$\bar{s}^O = T_{OP} \bar{s}^P$$

where $T_{OP} = T_{OI} T_{IG} T_{GP}$; $T_{IG} = T_{GI}^T$; $T_{GP} = T_{PG}^T$.

After substitution of \bar{r}^O and \bar{s}^O into (6) we obtain vector $\bar{\rho}^O$, represented in OCS. Unit vector, directed along the vector $\bar{\rho}^O$, определяется в виде

$$\bar{e}^O = \frac{1}{\rho} \begin{bmatrix} \rho_x & \rho_y & \rho_z \end{bmatrix}^T, \quad (9)$$

where $\rho = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}$ – distance between the SC center of mass and GDAS.

Unit vector \bar{e}^0 in BCS will be represented taking into account (3) and (9) in the form

$$\bar{e}^B = T_{BO} \bar{e}^0. \quad (10)$$

It is assumed in this article that antenna electric axis is directed along the axis minus $O_B Z_B$ BCD. Then unit vector $\bar{\xi}^B$, locating the antenna electric axis in BCS, will be represented in the following way

$$\bar{\xi}^B = |0 \quad 0 \quad -1|^T.$$

To find angle between unit vectors \bar{e}^B and $\bar{\xi}^B$ vector and scalar products are used

$$\sin \alpha = |\bar{e}^B \times \bar{\xi}^B|, \quad \cos \alpha = \bar{e}^B \cdot \bar{\xi}^B,$$

hence, value of the angle between antenna electric axis and direction towards the GDAS is defined by the equation

$$\alpha = \arctg(\sin \alpha / \cos \alpha).$$

Tracking the antenna electric axis directed to the GDAS, angle α will tend to zero.

Vector of the RW control moments

To generate control moment \bar{M}_u , created by RW when turning SC, it is suggested to use the following expression, represented in the BCS:

$$\bar{M}_u = \mu \cdot \bar{\xi} \times \bar{e} + \chi \cdot \bar{e} \times \bar{K} \cdot \dot{\bar{e}} - \eta \cdot \bar{e} \bar{e} \cdot \bar{\omega}_{bi}, \quad (11)$$

where $\mu = 0,08$; $\chi = 0,2$; $\eta = 2,0$ – coefficients of control law; $\bar{K} = \{k_{ij}\}$ – matrix of the control coefficients, where $i, j = 1, 2, 3$; $k_{ij} = 0,05$ with $i = j$; $k_{ij} = 0$ with $i \neq j$; $\bar{e} \bar{e}$ – dyad of the unit vector \bar{e} ; $\bar{\omega}_{bi}$ – vector of the absolute angular velocity of the SC.

Control law (11) has been previously used with Yuzhnoye-developed SC Sich-2 to provide the uniaxial attitude of the SC longitudinal axis in the Sun in standby mode.

Derivative of the unit vector $\dot{\bar{e}}$, included in (11), is defined in the form

$$\dot{\bar{e}} = (\bar{e}_i - \bar{e}_{i-1}) / \Delta t$$

where Δt – integration step of the differential equation system, describing the SC motion relatively the center of mass; \bar{e}_i, \bar{e}_{i-1} – value of the vector \bar{e} in the current and previous integration steps.

Control moment \bar{M}_u , obtained using the (11), is checked with the following expression

$$M_j = \begin{cases} M_j = M_{\max} & \text{при } M_j > M_{\max} \\ M_j & \text{при } M_j \leq M_{\max} \end{cases},$$

where $j = x, y, z$; M_{\max} – maximum value of the RW control moment.

Equation of SC motion relatively center of mass

SC motion relatively center of mass, as perfectly rigid body, is described with the following differential equations system:

– equation of motion, represented in vector-matrix form,

$$\dot{\bar{\omega}}_{bi} = (\bar{J})^{-1} \left(\bar{M}_\Sigma + \bar{M}_u + \bar{M}_{unload} - \bar{\omega}_{bi} \times (\bar{J} \bar{\omega}_{bi} + \bar{H}_\Sigma) \right), \quad (12)$$

where $\bar{\omega}_{bi}, \dot{\bar{\omega}}_{bi}$ – vectors of the absolute angular velocity and acceleration of the SC; \bar{J} – SC inertia tensor; \bar{H}_Σ – vector of RW kinetic momentum; \bar{M}_Σ – sum vector of external moments; \bar{M}_u – vector of RW control moments; \bar{M}_{unload} – vector of EM unloading moments;

– kinematic equation in the quaternary form

$$\dot{A}_{OB} = \frac{1}{2} \begin{bmatrix} 0 & -\bar{\omega}_{BO}^T \\ \bar{\omega}_{BO} & -\Phi(\bar{\omega}_{BO}) \end{bmatrix} \cdot A_{OB}, \quad (13)$$

where $A_{OB} = \{\lambda_o, \bar{\lambda}\}$ – normalized quaternion with scalar λ_o and vector $\bar{\lambda} = |\lambda_1 \quad \lambda_2 \quad \lambda_3|^T$ particles, which define the mutual orientation of OCS and BCS [2]; $\bar{\omega}_{BO}$ – vector of angular velocity of BCS relatively the OCS; $\Phi(\bar{\omega}_{BO})$ – skew-symmetric matrix of the vector $\bar{\omega}_{BO}$.

To determine the sum vector of the kinetic momentum \bar{H}_Σ , included in (12), the following expression is used

$$\bar{H}_\Sigma = \bar{H}_{i-1} - \bar{M}_u \cdot \Delta t,$$

where \bar{H}_{i-1} – sum vector of the RW kinematic momentum in the previous integration step.

After calculation of \bar{H}_Σ its value is verified in accordance with expression

$$H_j = \begin{cases} H_j = H_{\max} & \text{при } H_j > H_{\max} \\ H_j & \text{при } H_j \leq H_{\max} \end{cases},$$

where H_{\max} – maximum value of the RW kinetic momentum.

To calculate the vector of the unloading moments of \bar{M}_{unload} , included in (12), the following expression is used

$$\bar{M}_{\text{unload}} = \bar{L} \times \bar{B},$$

where \bar{L} – vector of EM magnetic moments; \bar{B} – vector of the Earth’s magnetic field (EMF) induction.

Vector \bar{L} is calculated from the formula

$$\bar{L} = \frac{\bar{K}_r}{B^2} \bar{B} \times \bar{H}_\Sigma,$$

where \bar{K}_r – diagonal matrix of coefficients.

According to the dipole model of the EMF, the expression for the vector of induction \bar{B} , represented in OCS, has the following form [3]

$$\bar{B} = \frac{m_g}{r^3} \begin{bmatrix} f_{12} \\ f_{22} \\ -2f_{32} \end{bmatrix},$$

where $m_g = 0,8 \cdot 10^{16}$ Weber · m – magnetic moment of the EMF dipole; f_{12}, f_{22}, f_{32} – matrix elements $T_{\text{OM}} = \{f_{ij}\}$; $T_{\text{OM}} = T_{\text{OI}} T_{\text{IM}}$; $T_{\text{IM}} = T_{\text{MI}}^T$; T_{MI} – matrix of transfer from ICS to the EMF dipole-coordinate system.

Sum vector \bar{M}_Σ in the equation (12) includes the following moments:

$$\bar{M}_\Sigma = \bar{M}_G + \bar{M}_m + \bar{M}_a + \bar{M}_s, \quad (14)$$

where $\bar{M}_G, \bar{M}_m, \bar{M}_a, \bar{M}_s$ – vectors of gravitation, magnetic, aerodynamic moments and moment of solar radiation force, respectively.

Vector of gravitation moment \bar{M}_G , included in (14), is defined by the expression

$$\bar{M}_G = \frac{3\mu}{r^3} \bar{k} \times \bar{J} \bar{k}$$

where $\mu = 0,3986 \cdot 10^{15}$ m³/s² – gravitation constant of the Earth; \bar{k} – unit vector, directed

along the radius-vector \bar{r} of the current point in the orbit.

Vector of magnetic moment \bar{M}_m , due to SC interaction with the EMF is defined by the expression

$$\bar{M}_m = (\bar{m} + \bar{K} \cdot \bar{B}) \times \bar{B},$$

where \bar{m} – vector of magnetic moments; \bar{K} – matrix of induction coefficients.

Vector of aerodynamic moment \bar{M}_a is defined by the expression

$$\bar{M}_a = (\bar{m}_a + \bar{c}_a \times (\bar{r}_{\text{c.m.}} / l_{\text{hull}})) A_{\text{hull}} l_{\text{hull}} q_a,$$

where \bar{m}_a – vector of torque coefficients; \bar{c}_a – vector of force factors; $\bar{r}_{\text{c.m.}}$ – radius-vector, connecting origin of O_d DCS with SC center of mass; q_a – velocity head; A_{hull} – SC characteristic area; l_{hull} – SC characteristic length.

Vector of the moment of solar radiation force \bar{M}_s is defined by the expression

$$\bar{M}_s = (\bar{m}_s + \bar{c}_s \times (\bar{r}_{\text{c.m.}} / l_{\text{hull}})) A_{\text{hull}} l_{\text{hull}} q_s,$$

where \bar{m}_s – vector of torque coefficients; \bar{c}_s – vector of force factors; $q_s = 2,36 \cdot 10^{-7}$ kgf/m² – «light» head.

Simulation results

Justification of using the law (11) to create the RW control moment when turning the SC to point the GDAS is shown with an example of SC dynamics numerical simulation, using the following initial data.

SC orbit is circular with altitude of 660 km and 98° inclination. Angles, which define the orbital initial position and Greenwich meridian relatively the vernal equinox direction: $\Omega = 0$; $\lambda_o = 0$. Geographical coordinates of GDAS: $\lambda_{\text{geo}} = 28,9^\circ$; $\varphi_{\text{geo}} = -48,9^\circ$.

SC inertial and center-of-mass characteristics:

$$\bar{J} = \begin{vmatrix} 16,4 & 1,6 & 0,10 \\ 1,6 & 14,6 & 2,5 \\ 1,0 & 2,5 & 17,1 \end{vmatrix} \text{ kg} \cdot \text{m}^2;$$

$$\bar{r} = |0,001 \quad -0,639 \quad -0,028|^T \text{ m.}$$

SC magnetic characteristics:

$$\overline{\overline{K}} = \begin{vmatrix} 9960 & -350 & 500 \\ -200 & 9490 & -100 \\ 200 & -100 & 8130 \end{vmatrix} \frac{\text{A} \cdot \text{m}^2}{\text{T}};$$

$$\overline{\overline{m}} = [-0,338 \quad -0,210 \quad 0,090]^T \text{A} \cdot \text{m}^2.$$

EM magnetic characteristics:

$$\overline{\overline{K_r}} = \begin{vmatrix} 0,05 & 0 & 0 \\ 0 & 0,05 & 0 \\ 0 & 0 & 0,05 \end{vmatrix};$$

$$L_u = 40 \text{ A} \cdot \text{m}^2.$$

RW characteristics:

$$H_{\max} = 12 \text{ N} \cdot \text{m} \cdot \text{s}; \quad M_{\max} = 0,24 \text{ N} \cdot \text{m}.$$

SC characteristic area and length:

$$A_{\text{hull}} = 1,3 \text{ m}^2; \quad l_{\text{hull}} = 1,55 \text{ m}.$$

Simulating the SC dynamics with the personal computer, the initial conditions were assumed corresponding to the standby mode:

$$\varphi(0) = 5^\circ; \quad \theta(0) = -5^\circ; \quad \psi(0) = 5^\circ;$$

$$\omega_x(0) = \omega_z(0) = 0,01^\circ / \text{c};$$

$$\omega_y(0) = 0,01^\circ / \text{c} + \omega_0,$$

where ω_0 – average orbital angular velocity.

Results of the numerical integration of the differential equation system (12), (13) are presented as the variation of the following parameters within the interval $t = 20$ min: angle α between vectors $\overline{\xi}^b$ and \overline{e}^b ; SC attitude angles in roll φ , pitch θ and yaw ψ ; SC angular velocities $\omega_x, \omega_y, \omega_z$; RW kinetic momenta h_x, h_y, h_z ; RW control moments M_x, M_y, M_z .

Simulation results are shown in Fig. 2–6.

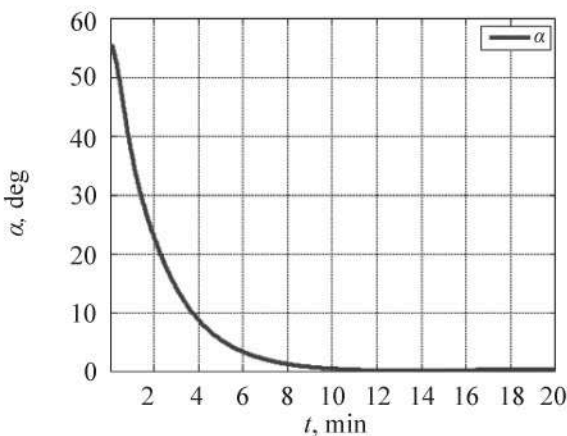


Рис. 2. Angle between the vectors $\overline{\xi}^b$ and \overline{e}^b

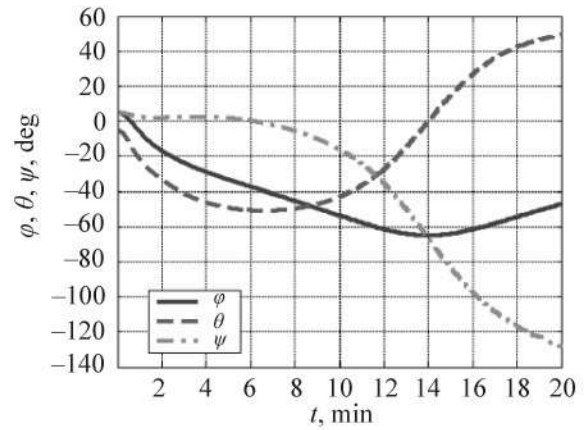


Fig. 3. SC attitude angles

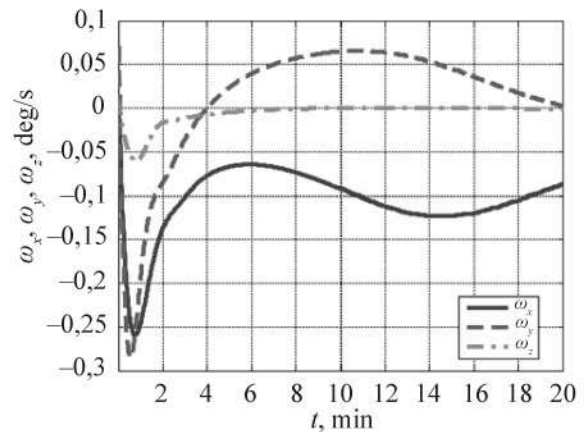


Fig. 4. SC angular velocities

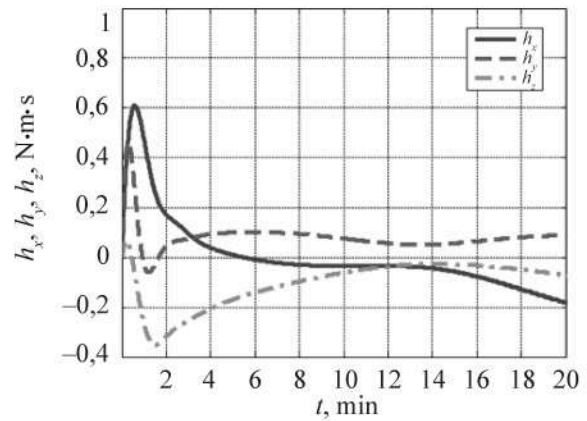


Fig. 5. Kinetic momentum of the RW

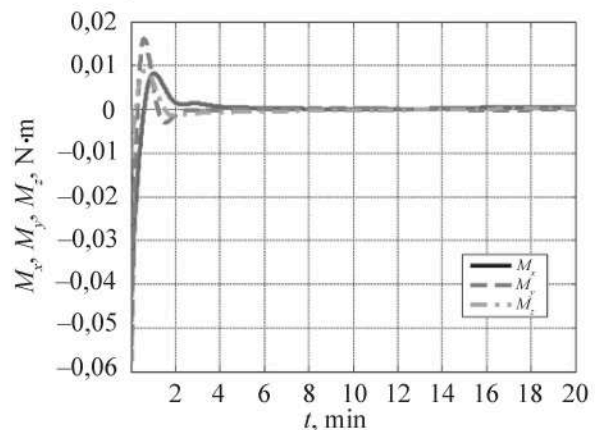


Fig. 6. RW control moment

It follows from Fig. 2–6 that during the time period $t = 10$ min the electric axis of the antenna is oriented towards the GDAS, with RW kinetic momenta not exceeding H_{\max} , and RW control moments do not exceed M_{\max} .

Conclusion

Results of SC dynamics simulation show that suggested control law of the reaction wheels provides the attitude of the electric axis fixed relatively the SC antenna body and pointed to the ground data acquisition station.

Suggested control law of the reaction wheels in addition to the considered case can be also applied when using antenna movable relatively the SC body in those orbit flight phases, which

require impermissibly high angular velocity of antenna turn to track the direction toward the GDAS.

List of references

1. Иванова Г. А., Остапчук С. В. Математическая модель магнитно-гравитационной системы ориентации для экспериментального микроспутника. *Космическая техника. Ракетное вооружение: Науч.-техн. сб.* 2009. С. 192 – 202.
2. Бранец В. Н., Шмыглевский И. П. Применение кватернионов в задачах ориентации твердого тела. М.: Наука, 1973. 320 с.
3. Проблемы ориентации искусственных спутников Земли. М.: Наука, 1966. 350 с.

Article поступила 10.08.2020